Symbols

`\e`	$\longrightarrow$	e
`\Eq`	$\longrightarrow$	E_q
`\Eqbar`	$\longrightarrow$	$\overline{E}_q$
`\Eqstar`	$\longrightarrow$	E_q^*
`\kfermi`	$\longrightarrow$	k_{F}
`\MN`	$\longrightarrow$	M_{N}
`\Mstar`	$\longrightarrow$	M_{N}^*
`\PiN`	$\longrightarrow$	Π_{N}
`\Piq`	$\longrightarrow$	Π_q
`\Piqtilde`	$\longrightarrow$	$\widetilde{\Pi}_q$
`\Pis`	$\longrightarrow$	Π_s
`\Pistilde`	$\longrightarrow$	$\widetilde{\Pi}_s$
`\Piu`	$\longrightarrow$	Π_u
`\Piutilde`	$\longrightarrow$	$\widetilde{\Pi}_u$
`\psibar`	$\longrightarrow$	$\overline{\psi}$
`\qbarq`	$\longrightarrow$	$\langle\overline{q}q\rangle_\rho$
`\qdaggerq`	$\longrightarrow$	$\langle q^\dagger q\rangle_\rho$
`\qdotu`	$\longrightarrow$	$q\cdot u$
`\qslash`	$\longrightarrow$	$\not{q}$
`\qtildeslash`	$\longrightarrow$	$\not{\tilde{q}}$

`\qvec`	$\longrightarrow$	$\mathbf{q}$
`\residue`	$\longrightarrow$	λ^2
`\residuebar`	$\longrightarrow$	$\overline{\lambda}^2$
`\rhoph`	$\longrightarrow$	ρ^{ph}
`\rhoth`	$\longrightarrow$	ρ^{th}
`\rhoN`	$\longrightarrow$	ρ_{N}
`\sigmav`	$\longrightarrow$	Σ_{v}
`\uslash`	$\longrightarrow$	$\not{u}$
`\utildeslash`	$\longrightarrow$	$\not{\tilde{u}}$
`\wzero`	$\longrightarrow$	ω_0
`\wzerobar`	$\longrightarrow$	$\overline{\omega}_0$

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Spectral asymmetries in nucleon sum rules at finite density

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Abstract

Apparent inconsistencies between different formulations of nucleon sum rules at finite density are resolved through a proper accounting of asymmetries in the spectral functions between positive- and negative-energy states.

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Relativistic hadronic models suggest that nucleons propagating in nuclear matter develop large, attractive scalar and repulsive vector self-energies [1]. Evidence supporting this picture in finite-density quantum chromodynamics (QCD) has emerged from some recent QCD sum rule calculations [2–5]. In particular, a rough connection is established between the nucleon scalar self-energy M_{N}^* and the scalar condensate $\langle\overline{q}q\rangle_\rho$ in medium, and between the nucleon vector self-energy Σ_{v} and the vector condensate $\langle q^\dagger q\rangle_\rho$. An alternative formulation of the sum rules [6], however, apparently gives qualitatively different results, in which the vector self-energy takes the opposite sign, so that the vector condensate induces attraction rather than repulsion. In this note, we resolve this ambiguity by properly accounting for spectral asymmetries in the finite-density sum rules for the nucleon, showing that the two formulations are each consistent with relativistic phenomenology.

The QCD sum rules for a nucleon in nuclear matter focus on a correlator of interpolating fields for the nucleon:

$$\Pi_{\mathrm{N}}(q) = i\int d^4x\, \mathrm{e}^{iq\cdot x}\, \langle\Phi_0|T\{\eta(x)\overline{\eta}(0)\}|\Phi_0\rangle\ , \tag{1}$$

where $|\Phi_0\rangle$ is the nuclear matter ground state, which is characterized by four-velocity u^μ and baryon density ρ_{N} in the rest frame. The interpolating field $\eta(x)$ is taken here and in Refs. [4,6] to be that advocated by Ioffe [7]. (See Ref. [8] for a treatment of more general interpolating fields and for explicit expressions.)

Because two four-vectors are available, the correlator can be decomposed into invariant functions in two ways:[1]

$$\begin{aligned}\Pi_{\mathrm{N}}(q) &= \not{q}\,\Pi_q(q^2, q\cdot u) + \Pi_s(q^2, q\cdot u)\\ &\qquad + \not{u}\,\Pi_u(q^2, q\cdot u)\\ &= \tilde{\not{q}}\,\widetilde{\Pi}_q(q^2, q\cdot u) + \widetilde{\Pi}_s(q^2, q\cdot u)\end{aligned} \tag{2}$$

[1]Relativistic covariance, time reversal, and parity imply that there are only three independent invariant functions [4].

$$+ \not{u}\, \widetilde{\Pi}_u(q^2, q \cdot u) \ , \tag{3}$$

where $\tilde{q}^\mu \equiv q^\mu - (q \cdot u)u^\mu$. The two sets of invariant functions are trivially related: $\widetilde{\Pi}_q = \Pi_q$, $\widetilde{\Pi}_s = \Pi_s$, and $\widetilde{\Pi}_u = (q \cdot u)\Pi_q + \Pi_u$. These two alternative decompositions will lead to the two sets of sum rules we consider.

The analytic structure of the correlator $\Pi_{\rm N}$, and consequently the invariant functions in Eqs. (2) and (3), is revealed by a standard Lehmann representation in energy ω, at fixed three-momentum $\mathbf{q}$ [9,4]. [We work for *convenience* in the rest frame of the matter, where $u^\mu = (1, \mathbf{0})$ and $q^\mu = (\omega, \mathbf{q})$.] The invariant functions are found to have singularities only on the real axis. In the upper half plane, $\Pi_{\rm N}(\omega, \mathbf{q})$ is equal to the retarded correlator and in the lower half plane to the advanced correlator. We will be concerned only with the discontinuities across the real axis (that is, the spectral functions), and therefore the infinitesimals that differentiate retarded, advanced, and time-ordered correlators for real ω are not relevant here.

We exploit the analytic structure of the correlator by considering integrals over contours running above and below the real axis, and then closing in the upper and lower half planes, respectively (see Ref. [4]). By approximating the correlator in the different regions of integration and applying Cauchy's theorem, we can derive a general class of sum rules, which manifest the duality between the physical hadronic spectrum and the spectral function calculated in a QCD expansion [10]:

$$\int_{-\overline{\omega}_0}^{\omega_0} W(\omega)\rho^{\rm ph}(\omega, \mathbf{q})\, d\omega - \int_{-\overline{\omega}_0}^{\omega_0} W(\omega)\rho^{\rm th}(\omega, \mathbf{q})\, d\omega = 0 \ . \tag{4}$$

Here $W(\omega)$ is a smooth (analytic) weighting function and the spectral densities $\rho^{\rm ph}$ and $\rho^{\rm th}$ are proportional to the discontinuities of the invariant functions across the real axis. (These sum rules can also be derived by expanding dispersion relations for retarded and advanced correlators with external frequency ω' in the limit $\omega' \to i\infty$.) The phenomenological spectral density $\rho^{\rm ph}$ models the low-energy physical spectrum, while the theoretical spectral density $\rho^{\rm th}$ is approximated using an operator product expansion (OPE) applied to $\Pi_{\rm N}$. The QCD sum rule approach assumes that, with suitable choices for W and the effective continuum

thresholds ω_0 and $-\overline{\omega}_0$, each integral can be reliably calculated and meaningful results extracted.

The thresholds ω_0 and $-\overline{\omega}_0$ act as effective boundaries beyond which the physical spectrum $\rho^{\rm ph}$, when moderately smoothed, is identical to that of the leading contributions to $\rho^{\rm th}$. At zero density, the discrete space-time symmetries imply that $\rho(-\omega, \mathbf{q}) = -\rho(+\omega, \mathbf{q})$, and so we are led to take $\overline{\omega}_0 = \omega_0$. That is, charge conjugation implies that the free-space spectral function corresponding to positive-energy states (like the nucleon) is equal in magnitude to that of the corresponding negative-energy states (like the antinucleon). Since the relative sign of the positive- and negative-energy spectral functions is negative, only a weighting function odd in ω yields a nonzero result in Eq. (4). In this case, we can convert the integral to one over ω^2 and then over $s = \omega^2 - \mathbf{q}^2$. The end result features only manifestly covariant integrals, as one would expect.

Useful choices for the weighting function at zero density are $W(\omega) = \omega \mathrm{e}^{-\omega^2/M^2}$, which generates the conventional Borel sum rules [11,4], or polynomials in ω, which generate the so-called finite energy sum rules (FESR) [10]. (Actually, ω times a monomial in s is used in the latter case, since $\rho = \rho(s)$ in vacuum.)

At finite density, ordinary charge conjugation symmetry is broken by the nonzero baryon number of the medium. This is clear from physical considerations: the propagation of a nucleon in ordinary nuclear matter is quite different from the propagation of an antinucleon. The consequence for the spectral function $\rho^{\rm ph}(\omega, \mathbf{q})$ is an asymmetry with respect to positive and negative ω. Asymmetries also appear naturally in $\rho^{\rm th}(\omega, \mathbf{q})$ by generalizing the OPE to finite density [see below]. As a result, we are also compelled to adopt an asymmetric duality interval so that $\omega_0 \neq \overline{\omega}_0$; this is the feature missed in Ref. [6].

Following Ref. [4], we saturate the phenomenological integral in Eq. (4) with a quasiparticle pole ansatz:

$$\Pi_{\rm N} \propto \frac{1}{\not{q} - \not{u}\Sigma_{\rm v} - M^*_{\rm N}} \ . \tag{5}$$

This implies

$$\begin{aligned}\rho_q^{\rm ph} &= \widetilde{\rho}_q^{\rm ph} \\ &= \frac{\pi}{2E_q^*}\left[\lambda^2\delta(\omega - E_q) - \overline{\lambda}^2\delta(\omega - \overline{E}_q)\right] \ , \end{aligned} \tag{6}$$

$$\begin{aligned}\rho_s^{\rm ph} &= \widetilde{\rho}_s^{\rm ph} \\ &= \frac{\pi}{2E_q^*}M_{\rm N}^*\left[\lambda^2\delta(\omega - E_q) - \overline{\lambda}^2\delta(\omega - \overline{E}_q)\right] \ , \end{aligned} \tag{7}$$

$$\rho_u^{\rm ph} = -\frac{\pi}{2E_q^*}\Sigma_{\rm v}\left[\lambda^2\delta(\omega - E_q) - \overline{\lambda}^2\delta(\omega - \overline{E}_q)\right] \ , \tag{8}$$

$$\widetilde{\rho}_u^{\rm ph} = \frac{\pi}{2E_q^*}E_q^*\left[\lambda^2\delta(\omega - E_q) + \overline{\lambda}^2\delta(\omega - \overline{E}_q)\right] \ , \tag{9}$$

where $E_q^* \equiv (q^2 + M_{\rm N}^{*2})^{1/2}$, $E_q = \Sigma_{\rm v} + E_q^*$, and $\overline{E}_q = \Sigma_{\rm v} - E_q^* < 0$. The notation is that ρ_i ($\widetilde{\rho}_i$) corresponds to the discontinuity of Π_i ($\widetilde{\Pi}_i$), with $i = \{q, s, u\}$. We have allowed for different residues λ^2 and $\overline{\lambda}^2$ for the positive and negative-energy poles, respectively. Different residues would naturally arise, for example, from the energy dependence of the self-energies. The form of the ansatz in Eq. (5) is otherwise constrained by Lorentz covariance. While a sharp quasinucleon state represents an extreme simplification of the actual spectrum, it is not unrealistic when smeared over energy scales of several hundred MeV, as we do in the sum rules. The physics motivation for the ansatz is discussed further in Ref. [4].

The operator product expansion for $\Pi_{\rm N}$ is developed in Refs. [4,12,8]. We keep only the terms from the leading diagrams here (corresponding to those considered in the first paper of Ref. [6]) to illustrate our point. For a quantitative analysis, one would include at least the contributions through terms involving four-quark condensates. The expansion is (suppressing parts that do not contribute to $\rho^{\rm th}$)

$$\begin{aligned}\Pi_{\rm N}(q) = \not q\Big[&-\frac{1}{64\pi^4}(q^2)^2\ln(-q^2) \\ &+ \frac{1}{3\pi^2}q\cdot u\ln(-q^2)\langle q^\dagger q\rangle_\rho\Big] \\ &+\frac{1}{4\pi^2}q^2\ln(-q^2)\langle\overline{q}q\rangle_\rho + \not u\frac{2}{3\pi^2}q^2\ln(-q^2)\langle q^\dagger q\rangle_\rho \ , \end{aligned} \tag{10}$$

which implies (specializing to the rest frame again)

$$\rho_q^{\rm th} = \widetilde{\rho}_q^{\rm th} = \pi[\theta(\omega - |\mathbf{q}|) - \theta(-\omega - |\mathbf{q}|)]$$

$$\times \left[\frac{1}{64\pi^4}(q^2)^2 - \frac{1}{3\pi^2}\omega\langle q^\dagger q\rangle_\rho\right] \ , \tag{11}$$

$$\rho_s^{\rm th} = \widetilde{\rho}_s^{\rm th} = -\pi[\theta(\omega - |\mathbf{q}|) - \theta(-\omega - |\mathbf{q}|)]$$

$$\times \left[\frac{1}{4\pi^2}q^2\langle\overline{q}q\rangle_\rho\right] \ , \tag{12}$$

$$\rho_u^{\rm th} = -\pi[\theta(\omega - |\mathbf{q}|) - \theta(-\omega - |\mathbf{q}|)] \left[\frac{2}{3\pi^2}q^2\langle q^\dagger q\rangle_\rho\right] \ , \tag{13}$$

$$\widetilde{\rho}_u^{\rm th} = \pi[\theta(\omega - |\mathbf{q}|) - \theta(-\omega - |\mathbf{q}|)]$$

$$\times \left[\frac{\omega}{64\pi^4}(q^2)^2 - (\frac{2}{3\pi^2}q^2 + \frac{1}{3\pi^2}\omega^2)\langle q^\dagger q\rangle_\rho\right] \ . \tag{14}$$

We now have all the ingredients needed to construct QCD sum rules using Eq. (4).

The phenomenological expectation from Relativistic Brueckner Hartree-Fock (RBHF) and mean-field calculations, as well as from Dirac phenomenology, is that at nuclear matter density $M_{\rm N}^* \approx 0.65M_{\rm N}$ and $\Sigma_{\rm v} \approx 0.35M_{\rm N}$, and the sum is relatively constant with density [1]. This expectation is supported by a simple version of the sum rules that is discussed in Ref. [4]. (A more complete sum rule is, however, not yet conclusive because of uncertainties from higher-order condensates [4,8].) In these rules, $M_{\rm N}^*$ is correlated with $\langle\overline{q}q\rangle_\rho$, and $\Sigma_{\rm v} > 0$ with $\langle q^\dagger q\rangle_\rho$.

The authors of two recent preprints, however, obtain very different qualitative results. In Ref. [6], the decomposition in Eq. (3) was (implicitly) adopted, and applied with $\mathbf{q} = 0$. The apparent advantage is that a direct projection onto the positive-energy quasinucleon is possible. That is, $\frac{1}{2}(1+\gamma_0)$ projects onto the positive-energy quasinucleon and $\frac{1}{2}(1-\gamma_0)$ projects onto the negative-energy quasinucleon. From these projected sum rules, $\langle q^\dagger q\rangle_\rho$ was found in Ref. [6] to be associated with *attraction* rather than repulsion, implying that $\Sigma_{\rm v} < 0$. The net result was a very large energy shift of the quasinucleon pole. Here we point out the missing elements in these calculations, which drastically affect the conclusions.

As noted above, the essential point is that the asymmetry in the phenomenological spectral density must necessarily be reflected in an asymmetry in the duality interval; that is, $\overline{\omega}_0 \neq \omega_0$. This point and its semi-quantitative implications are made clear by considering FESR's, using Eqs. (11)–(14) with monomials in ω as the weighting functions $W(\omega)$.

We start with the first decomposition, Eq. (2). Given a quasiparticle ansatz for the nucleon Eq. (5), this is a natural decomposition for isolating the scalar self-energy $M_{\rm N}^*$ (by considering Π_s/Π_q) and the vector self-energy $\Sigma_{\rm v}$ (by considering Π_u/Π_q). We apply Eq. (4) to each of the three functions ρ_q, ρ_s, and ρ_u, first with $W(\omega)=1$ and then with $W(\omega)=\omega$. We can derive further sum rules, of course, but these are sufficient to make our point. For simplicity and for a clear comparison to Ref. [6], we take $\mathbf{q}=0$ from here on, so that $q^2\to\omega^2$ in Eqs. (11)–(14). We obtain

$$q:\quad \frac{1}{2M_{\rm N}^*}(\lambda^2-\overline{\lambda}^2)=\frac{1}{320\pi^4}(\omega_0^5-\overline{\omega}_0^5)-\frac{1}{6\pi^2}\langle q^\dagger q\rangle_\rho(\omega_0^2+\overline{\omega}_0^2)\ , \tag{15}$$

$$\frac{1}{2M_{\rm N}^*}[M_{\rm N}^*(\lambda^2+\overline{\lambda}^2)+\Sigma_{\rm v}(\lambda^2-\overline{\lambda}^2)]=\frac{1}{384\pi^4}(\omega_0^6+\overline{\omega}_0^6)-\frac{1}{9\pi^2}\langle q^\dagger q\rangle_\rho(\omega_0^3-\overline{\omega}_0^3)\ , \tag{16}$$

$$s:\quad \frac{1}{2}(\lambda^2-\overline{\lambda}^2)=-\frac{1}{12\pi^2}\langle\overline{q}q\rangle_\rho(\omega_0^3-\overline{\omega}_0^3)\ , \tag{17}$$

$$\frac{1}{2}[M_{\rm N}^*(\lambda^2+\overline{\lambda}^2)+\Sigma_{\rm v}(\lambda^2-\overline{\lambda}^2)]=-\frac{1}{16\pi^2}\langle\overline{q}q\rangle_\rho(\omega_0^4+\overline{\omega}_0^4)\ , \tag{18}$$

$$u:\quad \frac{\Sigma_{\rm v}}{2M_{\rm N}^*}(\lambda^2-\overline{\lambda}^2)=\frac{2}{9\pi^2}\langle q^\dagger q\rangle_\rho(\omega_0^3-\overline{\omega}_0^3)\ , \tag{19}$$

$$\frac{\Sigma_{\rm v}}{2M_{\rm N}^*}[M_{\rm N}^*(\lambda^2+\overline{\lambda}^2)+\Sigma_{\rm v}(\lambda^2-\overline{\lambda}^2)]=\frac{1}{6\pi^2}\langle q^\dagger q\rangle_\rho(\omega_0^4+\overline{\omega}_0^4)\ . \tag{20}$$

One should be cautious about quantitative results extracted from these sum rules, but the qualitative features should persist in more sophisticated treatments.

The terms in the operator product expansion (OPE) odd in ω vanish at zero density. But at finite density, they imply, through duality, an asymmetry in the physical spectral functions, as manifested in Eq. (15). There are many more such terms as we extend the OPE to higher dimension.

If we expand in density, then from Eq. (15) we see that (recall that $\lambda^2=\overline{\lambda}^2$ and $\omega_0=\overline{\omega}_0$ at $\rho_{\rm N}=0$ from charge conjugation symmetry)

$$\omega_0-\overline{\omega}_0\sim O(\rho_{\rm N})\ , \tag{21}$$

$$\lambda^2-\overline{\lambda}^2\sim O(\rho_{\rm N})\ . \tag{22}$$

If we drop terms of higher order in $\rho_{\rm N}$, Eqs. (16), (18), and (20) become

$$\frac{1}{2}(\lambda^2+\overline{\lambda}^2) = \frac{1}{384\pi^4}(\omega_0^6+\overline{\omega}_0^6) \ , \tag{23}$$

$$\frac{1}{2}M_{\rm N}^*(\lambda^2+\overline{\lambda}^2) = -\frac{1}{16\pi^2}\langle\overline{q}q\rangle_\rho(\omega_0^4+\overline{\omega}_0^4) \ , \tag{24}$$

$$\frac{1}{2}\Sigma_{\rm v}(\lambda^2+\overline{\lambda}^2) = \frac{1}{6\pi^2}\langle q^\dagger q\rangle_\rho(\omega_0^4+\overline{\omega}_0^4) \ . \tag{25}$$

We can divide Eqs. (24) and (25) by (23) to find

$$M_{\rm N}^* = -\frac{24\pi^2}{\langle\omega_0^2\rangle}\langle\overline{q}q\rangle_\rho \qquad \text{and} \qquad \Sigma_{\rm v} = \frac{64\pi^2}{\langle\omega_0^2\rangle}\langle q^\dagger q\rangle_\rho \ , \tag{26}$$

where $\langle\omega_0^2\rangle \approx \frac{1}{2}(\omega_0^2+\overline{\omega}_0^2)$ to this order. Plugging in a typical continuum threshold [4,8], we find roughly the magnitudes, density dependence, and scalar-vector cancellations expected for the self-energies from relativistic phenomenology. Note that $\Sigma_{\rm v}$ is unambiguously positive. The scale of $\omega_0-\overline{\omega}_0$ is set by Eq. (15), which in conjunction with Eq. (17) and the results for $\Sigma_{\rm v}$ and $M_{\rm N}^*$ in Eq. (26), implies that $\omega_0-\overline{\omega}_0 \approx \Sigma_{\rm v}$.

Now we consider the second decomposition, Eq. (3), and extract the analogous sum rules. The rules from $\widetilde{\Pi}_q$ and $\widetilde{\Pi}_s$ are the same as above. The only different sum rules come from $\widetilde{\Pi}_u$:

$$\widetilde{u}: \quad \frac{1}{2}(\lambda^2+\overline{\lambda}^2) = \frac{1}{384\pi^4}(\omega_0^6+\overline{\omega}_0^6) - \frac{\langle q^\dagger q\rangle_\rho}{3\pi^2}(\omega_0^3-\overline{\omega}_0^3) \ , \tag{27}$$

$$\frac{\Sigma_{\rm v}}{2}(\lambda^2+\overline{\lambda}^2) + \frac{M_{\rm N}^*}{2}(\lambda^2-\overline{\lambda}^2) = -\frac{\langle q^\dagger q\rangle_\rho}{4\pi^2}(\omega_0^4+\overline{\omega}_0^4) + \frac{1}{448\pi^4}(\omega_0^7-\overline{\omega}_0^7) \ . \tag{28}$$

The underlying problem with Ref. [6] is made clear by Eq. (28). If $\overline{\omega}_0=\omega_0$ is assumed [and therefore $\overline{\lambda}^2=\lambda^2$ from Eq. (17)], one concludes that $\Sigma_{\rm v}<0$.

Adding the sum rules for $\widetilde{\Pi}_s=\Pi_s$ and $\widetilde{\Pi}_u$ (that is, making the $\frac{1}{2}(1+\gamma_0)$ projection), we obtain:

$$\lambda^2 = \frac{1}{384\pi^4}(\omega_0^6+\overline{\omega}_0^6) - \frac{\langle q^\dagger q\rangle_\rho}{3\pi^2}(\omega_0^3-\overline{\omega}_0^3) - \frac{\langle\overline{q}q\rangle_\rho}{12\pi^2}(\omega_0^3-\overline{\omega}_0^3) \ , \tag{29}$$

$$\lambda^2(\Sigma_{\rm v}+M_{\rm N}^*) = -\frac{\langle\overline{q}q\rangle_\rho}{16\pi^2}(\omega_0^4+\overline{\omega}_0^4) - \frac{\langle q^\dagger q\rangle_\rho}{4\pi^2}(\omega_0^4+\overline{\omega}_0^4) + \frac{1}{448\pi^4}(\omega_0^7-\overline{\omega}_0^7) \ . \tag{30}$$

Dividing these equations gives a sum rule for the positive-energy pole:

$$\begin{aligned} E_q(\mathbf{q}=0) &= \Sigma_{\rm v}+M_{\rm N}^* \\ &= \frac{-(\langle\overline{q}q\rangle_\rho+4\langle q^\dagger q\rangle_\rho)(\omega_0^4+\overline{\omega}_0^4)+\frac{1}{28\pi^2}(\omega_0^7-\overline{\omega}_0^7)}{\frac{4}{3}\left[\frac{1}{32\pi^2}(\omega_0^6+\overline{\omega}_0^6)-(\langle\overline{q}q\rangle_\rho+4\langle q^\dagger q\rangle_\rho)(\omega_0^3-\overline{\omega}_0^3)\right]} \ . \end{aligned} \tag{31}$$

This is the FESR analog of the Borel sum-rule equations derived in Ref. [6]. Again, if $\overline{\omega}_0 = \omega_0$ is assumed, $\langle q^\dagger q\rangle_\rho$ and the change with density of $\langle \overline{q}q\rangle_\rho$ both apparently reduce E_q from its zero density value of $M_{\rm N}$. However, once $\overline{\omega}_0 \neq \omega_0$ is allowed, several new terms contribute. Numerical estimates using $\omega_0 - \overline{\omega}_0 = \Sigma_{\rm v}$ from Eq. (26), condensate values from Ref. [4], and $\omega_0 \approx 1.6$–$1.7\,$GeV show that $E_q(\mathbf{q}=0)$ is essentially constant with density rather than decreasing precipitously, as found in Ref. [6].[2]

This quantitative result, which involves various cancellations, is very sensitive to details of the calculation: the choice of weighting function, the continuum threshold, the number of terms kept in the OPE. In contrast, sum rules derived from the first decomposition of $\Pi_{\rm N}$, which lead to separate expressions for the scalar and vector self-energies rather than for the sum, are more robust. In Refs. [4,8], the decomposition of Eq. (2) was considered, with an asymmetric weighting function used to suppress contributions from the negative-energy quasinucleon in favor of the positive-energy quasinucleon. The possibility $\omega_0 \neq \overline{\omega}_0$ was not considered in these references; however, explicit calculations show that making this generalization is numerically unimportant. Thus there is a qualitative difference in the sensitivity of the two correlator decompositions [Eqs. (2),(3)] to the (uncertain) details of the continuum contribution.

In summary, we have reexamined the results of Ref. [6], which implied that certain QCD sum rules predict that the vector self-energy of a nucleon in nuclear matter is attractive, rather than repulsive as implied by relativistic phenomenology and other sum rules [4]. By properly accounting for asymmetries in the spectral functions between positive- and negative-energy states, we have shown that there is no inconsistency, and that all formulations lead to a repulsive vector interaction.

[2]These rather high values for ω_0 are needed in the FESR at $\rho_{\rm N} = 0$ to obtain a sensible result for $M_{\rm N}$. In addition, we note that the sum rules imply that $\omega_0 + \overline{\omega}_0$ is roughly constant with density.

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